

### (EQUATION) [B-1-3]

- Mathematically, a pressure gradient,  $\nabla p$  ( $\text{grad } p$ ), is a scalar transformed to a vector with:  
**Magnitude** = maximum rate of change of pressure ( $p$ ) per unit length of the coordinate space at the given point  
**Direction** = direction of the maximum rate of change of pressure ( $p$ ) at the given point
- Using the vector quantity of the pressure gradient, directional change of pressure (commonly understood as "**pressure gradient in the direction of the flow**") is given by:  
$$\frac{dp}{ds} = \nabla p \cdot d\hat{s}$$

(This represents "**how much change of pressure takes place**" in the direction specified by the unit vector  $d\hat{s}$  in the tangential direction of a curved "streamline" coordinate  $s$ ).
- "Favorable"** pressure gradient: means that the pressure **decreases** in the direction of flow:  
$$\left(\frac{dp}{ds} < 0\right)$$
- "Adverse"** pressure gradient: means that the pressure **increases** in the direction of the flow:  
$$\left(\frac{dp}{ds} > 0\right)$$
- In general, a basic mathematical operator to represent a vector field is the "**del**" operator:  
$$\nabla = \text{gradient} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) = \hat{i} \frac{\partial}{\partial x} ( ) + \hat{j} \frac{\partial}{\partial y} ( ) + \hat{k} \frac{\partial}{\partial z} ( )$$
  
$$\nabla = \text{gradient} = \left(\frac{\partial}{\partial r}, \frac{1}{r} \frac{\partial}{\partial \theta}, \frac{\partial}{\partial z}\right) = \hat{e}_r \frac{\partial}{\partial r} ( ) + \hat{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} ( ) + \hat{e}_z \frac{\partial}{\partial z} ( )$$
- In 3-D coordinates, pressure gradient can be expressed either in rectangular (Cartesian) or cylindrical coordinate system as:  
$$\nabla p = \text{grad}(p) = \frac{\partial p}{\partial x} \hat{i} + \frac{\partial p}{\partial y} \hat{j} + \frac{\partial p}{\partial z} \hat{k}$$
  
$$\nabla p = \text{grad}(p) = \frac{\partial p}{\partial r} \hat{e}_r + \frac{1}{r} \frac{\partial p}{\partial \theta} \hat{e}_\theta + \frac{\partial p}{\partial z} \hat{e}_z$$

Lined area for notes, consisting of multiple horizontal dashed lines.